$$\Gamma[i,1] = \Gamma[i,0] + \left.\frac{\partial \Gamma}{\partial z}\right|_{i,0} h_z + \left.\frac{\partial^2 \Gamma}{\partial z^2}\right|_{i,0} \frac{h_z^2}{2}.$$

The value of the second derivative is found directly from the equation for Γ extended to the boundary. A 40 × 40 grid uniform over z and nonuniform over r was used for the solution in the central region. The r-variation of the computing step was specified in the following way:

$$r[i] = h_r[0] \frac{(1+\alpha)^i - 1}{\alpha} \quad \text{or} \quad h_r[i+1] = (1+\alpha) h_r[i].$$

For the computations discussed above $\alpha = 0.024$.

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SECONDARY FLOWS BESIDE A CYLINDER IN A COMPLEX SOUND FIELD

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It is known that steady flows arise beside a solid surface in the presence of a sound field which can to a certain extent exert an effect on the processes of heat and mass exchange [1-3]. As a rule, all papers from this area refer to the case in which one can represent the sound field in the form of a single wave. However, situations are often encountered in practice in which the sound field is complex; i.e., it consists of several vibrations whose amplitudes and frequencies are unlike in the general case. The secondary flows which form beside a circular cylinder placed in a complex soundfield are investigated in this paper.

Let n plane waves with the following parameters encounter a circular cylinder of radius R: A_n is the velocity amplitude of the acoustic shift in the n-th wave, ω_n is the frequency, a_n is the point of encounter of the wave with the cylinder, and φ_n is the phase of the wave. Let us consider the case in which the radius of the cylinder is significantly less than the wavelength; then the flow beside the cylinder can be treated as incompressible.

The Navier–Stokes equation describing the motion of a viscous incompressible liquid has the form

$$\frac{\partial}{\partial t}(\nabla^2\psi) - \varepsilon \frac{\partial(\psi, \nabla^2\psi)}{\partial(\theta, r)} = \frac{1}{2} H^2 \nabla^4 \psi, \tag{1}$$

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where $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{1+r}\frac{\partial}{\partial r} + \frac{1}{(1+r)^2}\frac{\partial^2}{\partial\theta^2}$; $\partial(\psi, \nabla^2\psi)/\partial(\theta, r)$ is the Jacobian determinant; $\varepsilon = s_1/R$; $H = \delta_{ac}/R$; $\delta_{ac} = (2\nu/\omega_1)^{1/2}$; and s_1 is the amplitude of the acoustic shift in the first wave. The stream function ψ is defined as

$$u = \partial\psi/\partial r,\ v = -1/(1 + r)\cdot\partial\psi/\partial\theta. \tag{2}$$

The boundary conditions are of the form

$$\psi = \partial\psi/\partial r = 0 \ \text{at}\ r = 0;$$
$$\psi = (1 + r)\sum_{k=1}^{n} B_k \sin(\theta - a_k)\, e^{i\left(b_k^2 t + \varphi_k\right)} \ \text{as} \ r \to \infty, \tag{3}$$

where $B_k = A_k/A_1$ and $b_k = (\omega_k/\omega_1)^{1/2}$.

Equations (1)-(3) are written in the following dimensionless variables:

$$r = (\tilde{r} - R)/R,\ \psi = \tilde{\psi}/A_1R,\ t = \tilde{t}\omega_1. \tag{4}$$

Let us consider the case in which the following conditions are fulfilled:

$$\varepsilon \ll 1; \tag{5}$$
$$H \ll 1. \tag{6}$$

We will solve the problem by using the method of spliced asymptotic expansions [4, 5]. Let us divide the entire region occupied by the liquid into two regions: an interior one (with a characteristic size δ_{ac} in the direction perpendicular to the cylinder surface) and an exterior one (with a characteristic size R). The exterior variables are defined by Eq. (4), and the interior ones are written in the form

$$\eta = (\tilde{r} - R)/\delta_{ac},\ m = \tilde{\psi}/A_1\delta_{ac},\ t = \tilde{t}\omega_1. \tag{7}$$

The relation between the interior and exterior variables has the form

$$r = H\eta,\ \psi = Hm, \tag{8}$$

where η and r are measured from the cylinder surface and θ is measured from the encounter point of the first wave (A_1).

Let us consider the exterior region. In view of condition (5) the solution will be sought by the method of successive approximations:

$$\psi = \psi^{(0)} + \varepsilon\psi^{(1)} + O(\varepsilon^2). \tag{9}$$

Having substituted (9) into (1) and collected terms with identical powers of ε, we obtain

$$\frac{\partial}{\partial t}(\nabla^2\psi^{(0)}) = \frac{1}{2}H^2\nabla^4\psi^{(0)},\ \frac{\partial}{\partial t}(\nabla^2\psi^{(1)}) - \frac{\partial(\psi^{(0)}, \nabla^2\psi^{(0)})}{\partial(\theta, r)} = \frac{1}{2}H^2\nabla^4\psi^{(1)}. \tag{10}$$

Although Eqs. (10) are linear, their solutions, written with the use of Hankel functions, are, however, rather awkward, which hinders subsequent analysis. Therefore, we represent $\psi^{(i)}$ in the form

$$\psi^{(i)} = \psi^{(i0)} + H\psi^{(i1)} + O(H^2),\ i = 0, 1\ldots, \tag{11}$$

by use of the condition (6). Having substituted (11) into (10) and collected terms with identical powers of H, we obtain

$$\begin{gathered}\frac{\partial}{\partial t}(\nabla^2\psi^{(00)}) = 0,\ \frac{\partial}{\partial t}(\nabla^2\psi^{(01)}) = 0, \ldots,\\ \frac{\partial}{\partial t}\left(\nabla^2\psi_H^{(10)}\right) - \left\{\frac{\partial(\psi^{(00)}, \nabla^2\psi^{(00)})}{\partial(\theta, r)}\right\}_H = 0,\\ \frac{\partial}{\partial t}\left(\nabla^2\psi_H^{(11)}\right) - \left\{\frac{\partial(\psi^{(00)}, \nabla^2\psi^{(01)})}{\partial(\theta, r)} + \frac{\partial(\psi^{(01)}, \nabla^2\psi^{(00)})}{\partial(\theta, r)}\right\}_H = 0,\\ \left\{\frac{\partial(\psi^{(00)}, \nabla^2\psi^{(02)})}{\partial(\theta, r)} + \frac{\partial(\psi^{(01)}, \nabla^2\psi^{(01)})}{\partial(\theta, r)} + \frac{\partial(\psi^{(02)}, \nabla^2\psi^{(00)})}{\partial(\theta, r)}\right\}_{st} = \frac{1}{2}\nabla^4\psi_{st}^{(10)}.\end{gathered} \tag{12}$$

Using the second boundary condition (3), one can show that the function $\psi^{(0j)}$ does not contain a component which is time-independent; therefore, Eqs. (12) will take the form

$$\begin{gathered}\nabla^2\psi^{(0j)} = 0,\ \ j = 0, 1, 2\ldots;\\ \frac{\partial}{\partial t}\left(\nabla^2\psi_H^{(1j)}\right) = 0;\ \ \nabla^4\psi_{st}^{(10)} = 0,\end{gathered} \tag{13}$$

where $\psi^{(10)} = \psi_{st}^{(10)}(\theta, r) + \psi_H^{(10)}(\theta, r, t)$.

For simplicity we will consider the sound field subsequently to consist of two plane waves. Then the solutions of Eqs. (13) should satisfy the following boundary conditions:

$$\psi^{(0j)} = \begin{cases} (1+r)\,[\sin\theta e^{it} + B\sin(\theta - a)\, e^{i(b^2t+\varphi)}], & j = 0, \\ \text{bounded.} & j \neq 0, \end{cases} \tag{14}$$

$\psi^{(10)}$ = bounded as $r \to \infty$, and the exterior solution should be asymptotically spliced with the interior solution as $r \to 0$; i.e.,

$$\psi(0) \cong Hm(\infty). \tag{15}$$

Let us consider the interior region. To this end let us rewrite Eq. (1) in the interior variables (7)

$$\frac{\partial}{\partial t}(\nabla^2 m(\eta, \theta, t, H)) - \varepsilon \frac{\partial\left(m(\eta, \theta, t, H), \nabla^2 m(\eta, \theta, t, H)\right)}{\partial(\theta, \eta)} = \frac{1}{2}\nabla^4 m(\eta, \theta, t, H). \tag{16}$$

Similarly to the exterior solution, we will seek the solution in the interior region in the form of a series:

$$m = m^{(00)} + O(H) + \varepsilon\left[m_{st}^{(10)} + m_{H}^{(10)} + O(H)\right] + O(\varepsilon^2). \tag{17}$$

It has been shown in [5] that an expansion of the type (11) and (17) can be applied only in the case in which the Reynolds number calculated from the velocity of the steady secondary flow is small ($Re_{st} = A^2/\omega\nu \ll 1$). The case of $Re_{st} \gg 1$ will be discussed separately.

Having substituted (17) and performed the same operations as in the derivation of Eqs. (13), we obtain

$$m_{\eta\eta t}^{(00)} - \frac{1}{2} m_{\eta\eta\eta\eta}^{(00)} = 0; \tag{18a}$$

$$m_{st\ \eta\eta\eta\eta}^{(10)} = 2\langle m_{\eta}^{(00)} m_{\eta\eta\theta}^{(00)} - m_{\theta}^{(00)} m_{\eta\eta\eta}^{(00)}\rangle. \tag{18b}$$

The functions m^{ij} should satisfy the boundary conditions

$$m^{(ij)} = \partial m^{(ij)}/\partial\eta = 0 \quad \text{at} \quad \eta = 0,\ i, j = 0, 1\ldots, \tag{19}$$

and be asymptotically spliced to the exterior solution.

The solutions of the first equation of (13) and of Eq. (18a) which satisfy the conditions (14), (15), and (19) are of the form

$$\psi^{(00)} = \left[1 + r - \frac{1}{1+r}\right] \text{Real}\,[\sin\theta \cdot e^{it} + B\sin(\theta - a) \cdot e^{i(b^2t+\varphi)}], \tag{20}$$

$$m^{(00)} = 2\,\text{Real}\,[\sin\theta \cdot \xi_1(\eta)\, e^{it} + Bb^{-1}\sin(\theta - a) \cdot \xi_1(b\eta)\, e^{i(b^2t+\varphi)}] \tag{21}$$

for the exterior and interior regions, respectively, where

$$\xi_1(\eta) = \eta + \frac{1-i}{2}[e^{-(1+i)\eta} - 1].$$

Prior to starting the search for a steady component of the stream function, we note that the right-hand side in Eq. (18b) will have a different analytic form depending on the relation between the frequencies of the vibrations of the two waves.

Let us consider the case of unlike frequencies ($b \neq 1$). Using (21) and having calculated the average terms, Eq. (18b) is written in the form

$$m_{st\ \eta\eta\eta\eta}^{(10)} = 2\sin 2\theta \cdot f_1(\eta) + 2B^2 b\sin 2(\theta - a) \cdot f_1(b\eta), \tag{22}$$

where

$$f_1(\eta) = e^{-(1+i)\eta} + e^{-(1-i)\eta} + i\eta e^{-(1+i)\eta} - i\eta e^{-(1-i)\eta} - 2e^{-2\eta}. \tag{23}$$

The solutions of the third equation of (13) and of Eq. (22) which satisfy the conditions (14), (15), and (19) are of the form

$$m_{st}^{(10)} = \Phi_1(\eta)\sin 2\theta + B^2 b^{-3}\Phi_1(b\eta)\sin 2(\theta - a); \tag{24}$$

$$\psi_{st}^{(10)} = \frac{3}{4}\left[\frac{1}{(1+r)^2} - 1\right][\sin 2\theta + B^2 b^{-3}\sin 2(\theta - a)] \tag{25}$$

for the interior and exterior regions, respectively, where

$$\Phi_1(\eta) = \frac{13}{4} - \frac{3}{2}\eta - \frac{3+2i}{2}e^{-(1+i)\eta} - \frac{3-2i}{2}e^{-(1-i)\eta} + \frac{i}{2}\eta e^{-(1-i)\eta} - \frac{i}{2}\eta e^{-(1+i)\eta} - \frac{1}{4}e^{-2\eta}. \tag{26}$$

It follows from Eqs. (24) and (25) that in the case of unlike frequencies the steady flow is a superposition of secondary flows corresponding to each vibration separately.

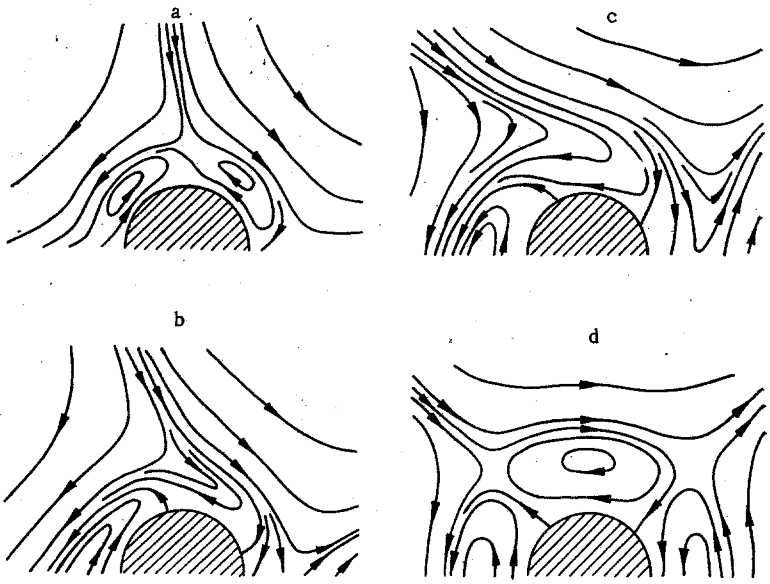

Fig. 1

The streamlines of the steady flow in the interior region are shown in Fig. 1 for a variation of the relative amplitude B from 0.05 to 10.0 ($\omega_1 = 4\omega_2$, $a = \pi/4$). It is evident that the steady flow for small values of B (Fig. 1a, B = 0.05) is determined by the first vibration and resembles in its nature the flow described by Schlichting [6]. As the amplitude of the second vibration increases, the nature of the flow is significantly altered. One can note those situations in which closed streamlines are absent in the interior region (Fig. 1b and c where B = 0.4 and 0.8). Upon a further increase in B the nature of the steady flow is determined by the parameters of the second vibration (Fig. 1d, B = 10.0).

Thus, the structure of the steady secondary flow in the case under discussion is more complex than in the Schlichting problem. However, one can predict the nature of the flow without resorting to the aid of a computer but investigating only the position of the branch lines. By branch line is understood a streamline at whose intersection the tangential component of the velocity field of the steady flow changes sign.

Thus, near the surface one can, by expanding (24) into a Taylor series and restricting oneself to a quadratic term, derive the following expression for determination of the angular coordinate of the branch line of the interior flow (the interior branch line):

$$\theta = \frac{1}{2} \operatorname{arctg} \left[\frac{B^2 b^{-1} \sin 2a}{1 + B^2 b^{-1} \cos 2a} \right] + \frac{\pi n}{2}, \quad n = 0, 1 \ldots. \tag{27}$$

Similarly, we obtain a relation from (25) for determination of the angular coordinate of the branch line of the exterior flow (the exterior branch line)

$$\theta = \frac{1}{2} \operatorname{arctg} \left[\frac{B^2 b^{-2} \sin 2a}{1 + B^2 b^{-2} \cos 2a} \right] + \frac{\pi n}{2}, \quad n = 0, 1 \ldots. \tag{28}$$

It is evident from a comparison of (27) and (28) that the exterior and interior branch lines do not coincide with each other, while they do coincide in the Schlichting problem. Coincidence of the exterior and interior branch lines will be observed in our case either when B = 0 or ∞, which corresponds to a simple sound field, or when $a = \pi n/2$. Consequently, the structure of the flow with these parameters will be the same as in [6]. We note that the pattern of the steady flow is repeated after $\pi/2$.

Let us consider the case of identical frequencies (b = 1). Then Eq. (18b) is written in the form

$$m^{(10)}_{st\,\eta\eta\eta\eta} = 2L(\theta) f_1(\eta) + 4B \sin a \cdot \sin \varphi \cdot f_2(\eta), \tag{29}$$

where

$$L(\theta) = \sin 2\theta + B^2 \sin 2(\theta - a) + 2B \sin (2\theta - a) \cdot \cos \varphi; \tag{30}$$

$f_2(\eta) = 2e^{-2\eta} - e^{-(1+i)\eta} - e^{-(1-i)\eta} + \eta e^{-1(1+i)\eta} + \eta e^{-(1-i)\eta}$, and $f_1(\eta)$ is defined by Eq. (23).

The solutions of the third equation of (13) and of Eq. (29) which satisfy the conditions (14), (15), and (19) are of the form

$$m_{st}^{(10)} = L(\theta)\Phi_1(\eta) + B\sin a \cdot \sin\varphi \cdot \Phi_2(\eta); \quad (31)$$

$$\psi_{st}^{(10)} = \frac{3}{4}\left[\frac{1}{(1+r)^2} - 1\right] \cdot L(\theta) - 3B\ln(1+r) \cdot \sin a \cdot \sin\varphi, \quad (32)$$

where $\Phi_2(\eta) = 3/2 - 3\eta - (1-2i)e^{-(1+i)\eta} - (1+2i)e^{-(1-i)\eta} - \eta e^{-(1+i)\eta} - \eta e^{-(1-i)\eta} + 1/2 e^{-2\eta}$, and $\Phi_1(\eta)$ and $L(\theta)$ are defined by Eqs. (26) and (30), respectively.

Thus, superposition of steady flows is not observed in the case of identical frequencies, since a contribution produced by the nonlinear interaction of two vibrations in the interior region appears in Eqs. (31) and (32).

The streamlines of the steady flow in the interior (left) and exterior (right) regions are shown in Fig. 2 for a variation of the difference in the phases φ between the two vibrations ($\omega_1 = \omega_2$, $a = \pi/2$, $A_1 = A_2$). It is evident that when the vibrations occur in phase, $\varphi = 0$ (Fig. 2a), then the nature of the flow both in the exterior and the interior regions completely coincides with the Schlichting flow [6]. Subsequently, the pattern is qualitatively altered (as φ increases): In the interior region neighboring vortices recede from the surface of the cylinder and decrease in size right down to complete disappearance (Fig. 2b, $\varphi = \pi/6$); the upper vortex also decreases in size, but it is pressed to the surface of the cylinder. Two closed vortices appear in the exterior region whose size decreases as φ increases; at the same time a large-scale circulating flow develops in the exterior and interior regions. We note that when $\varphi = 90°$ (Fig. 2c) the pattern obtained agrees in nature with the flow described by Longuet and Higgins [7], who treated this particular case to describe anomalous oceanic currents which form around isolated islands. It has been shown in [8] that the nature of the Longuet–Higgins flow does not depend on the number Re_{st} and is a peculiar analog of a Poiseuille flow.

Let us determine the position of the branch lines. Having repeated similar calculations as in the derivation of Eqs. (27) and (28), we obtain

$$\theta = \frac{1}{2}\arcsin\left(\pm\frac{E}{C}\cos\gamma\right) + \frac{\gamma}{2} + \pi n, \quad (33)$$

where the plus sign refers to the interior branch line and the minus sign refers to the exterior one, and where

$$E = B\sin\varphi \cdot \sin a; \quad C = \frac{1}{2} + B\cos\varphi \cdot \cos a + \frac{1}{2}B^2\cos 2a;$$

$$\gamma = \operatorname{arctg}\frac{D}{C} + \pi n; \quad D = B\cos\varphi \cdot \sin a + \frac{1}{2}B\sin 2a.$$

It follows from Eq. (33) that the flow pattern repeats in π. In addition both the interior and exterior branch lines alternate not in $\pi/2$, as is observed for the case of unlike frequencies, but their spatial separation depends on the parameters which characterize the complex sound field. We note that such situations may be realized when the exterior and interior branch lines coincide: $B = 0$ or ∞, which corresponds to a simple sound field; $a = \pi n$, i.e., the propagation lines of the two waves coincide; and $\varphi = \pi n$, i.e., the vibrations occur either in phase or out of phase. Consequently, the nature of the flow in these cases agrees completely with the Schlichting flow [6]. One should also note that if $|E/C \cos\gamma| > 1$, then no branch lines at all exist and the steady flow is a large-scale circulation (a flow of this type is shown in Fig. 2c). However, if $B = 1$, $a = \pi$, $\varphi = 0$, then there is no steady flow. This is physically confirmed, since the case in question corresponds to the placement of the cylinder at a velocity node of a standing wave.

Since a large-scale circulating flow appears near the cylinder in the case of identical frequencies, a steady moment of forces acts on the cylinder which is determined in the form

$$M = 4\pi\mu R(L/\Delta_{ac})(A_1A_2/\omega)\sin a \cdot \sin\varphi.$$

One can convince oneself that the time-independent moment of the forces is equal to zero in the case in which a complex sound field reduces to a simple one, i.e., to the Schlichting problem [6]. As calculations have shown, no steady force at all acts on the cylinder in the case $\omega_1 \neq \omega_2$.

The results presented above are described in Eulerian variables. However, the experimental investigation of secondary flows are conducted, as a rule, with the use of labeled particles (the tracking method), whose behavior is described in Lagrangian variables. If the Eulerian and Lagrangian descriptions give identical results for a steady flow, then they differ in the case of nonsteady motion; i.e., the streamlines (Eulerian variables) do not coincide with the particle trajectories (Lagrangian variables). It has been shown in [9] that the trajectory of a particle is related to the streamline of the secondary flow by the equation

$$\psi_{st}^{L}\mathbf{k} = \psi_{st}^{E}\mathbf{k} + F\mathbf{k},$$

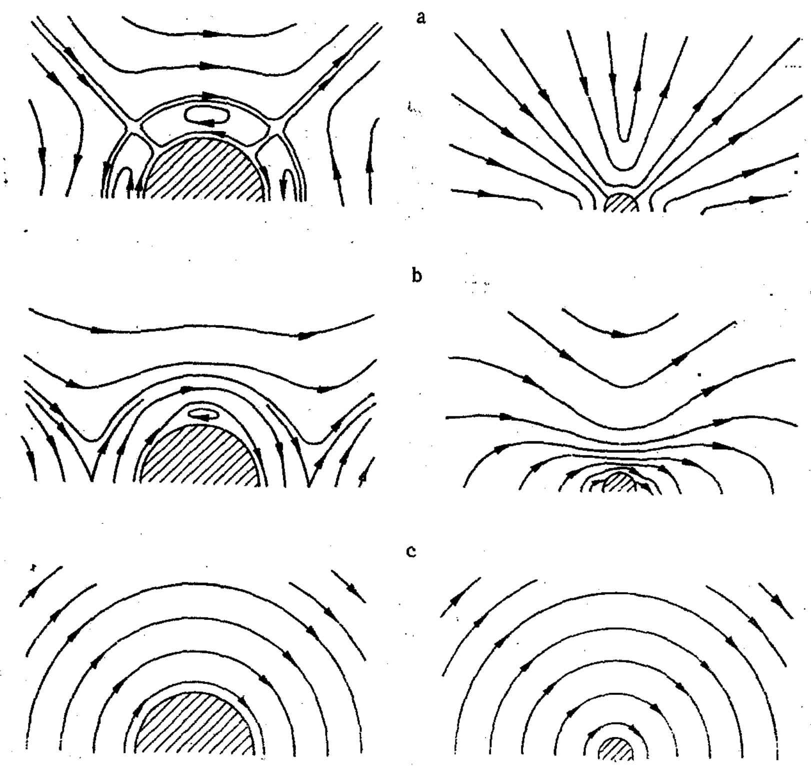

Fig. 2

where

$$Fk = -\frac{1}{2}\left\langle \left(\int_0^t u_H^E dt \right) \times u_H^E \right\rangle;$$

u_H^E is the pulsation velocity in Eulerian variables and **k** is the unit vector in the z direction.

By using Eqs. (20) and (21), we obtain in the case of unlike frequencies

$$F = O(H); \quad F = -\frac{1}{2}\sin 2\theta \cdot \Phi_3(\eta) - \frac{4}{2}B^2 b^{-3}\sin 2(\theta - a)\cdot\Phi_3(b\eta) + O(H) \qquad (34)$$

in the exterior and interior regions, respectively, where

$$\Phi_3(\eta) = 1 + e^{-2\eta} - e^{-(1+i)\eta} - e^{-(1-i)\eta} - i\eta e^{-(1+i)\eta} + i\eta e^{-(1-i)\eta}. \qquad (35)$$

It follows from the first equation of (34) that the streamlines and particle trajectories coincide with an accuracy of to terms of the order O(H), which is in agreement with the case of a simple sound field [9].

In the case of identical frequencies we obtain

$$F = -\frac{1}{2}B\left[1 - \frac{1}{(1+r)^4}\right]\sin\varphi \cdot \sin a + O(H),$$

$$F = -\frac{1}{2}L(\theta)\Phi_3(\eta) - \frac{1}{2}B\sin\varphi \cdot \sin a \cdot \Phi_4(\eta) + O(H)$$

for the exterior and interior regions, respectively, where $\Phi_3(\eta)$ and $L(\theta)$ are defined by Eqs. (35) and (30) and

$$\Phi_4(\eta) = 2 - 4\eta + 2e^{-2\eta} - 2e^{-(1+i)\eta} - 2e^{-(1-i)\eta} + 2\eta e^{-(1+i)\eta} - 2\eta e^{-(1-i)\eta}.$$

Thus, the streamlines and particle trajectories do not now coincide in the exterior region in the case of identical frequencies. However, as supplementary calculations on a computer have shown, the nature of the flow in Lagrangian variables does not differ significantly from the Eulerian description either in the case of identical or unlike frequencies.

The results presented above are obtained on the assumption that the Reynolds number calculated from the velocity of the secondary flows is small. When $Re_{st} \geq 1$, the equation which describes a steady flow in the exterior region is of the form [5]

$$\frac{\partial(\psi_{st}, \nabla^2\psi_{st})}{\partial(\theta, r)} = \frac{1}{\mathrm{Re}_{st}} \nabla^4\psi_{st}. \qquad (36)$$

A steady flow in the interior region is described as before by Eq. (18b). It is evident that although a superposition of the secondary flows characteristic of each vibration is accomplished in the interior region in the case of unlike frequencies, no superposition takes place, however, in the exterior region when $\mathrm{Re}_{st} \geq 1$ due to the nonlinearity of Eq. (36).

We note in conclusion that the superposition of secondary flows both of the interior and exterior regions is not performed in the higher-order terms [for example, $O(\varepsilon^3)$], independently of the size of Re_{st} and the relation between the frequencies.

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